Spectral Analysis Of Digital Filter Tuned For Mechanical Resonant Frequency Reduction In Multi-Mass Mechanical Systems In Electrical Direct Drive

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Abstract — This paper presents a mathematical model of direct drive system. Model was presented in state-space and transfer function representation. Includes a four-mass structure for the mechanical part. Article presents nonparametric identification of mechanical resonance frequency using two signals: PRBS (pseudo random binary sequence) and chirp. The result was used to tune digital notch filter for mechanical resonance attenuation. The filter is inserted between the speed controller and current controller. The article discusses the method of determining the delay caused by the filter. Knowledge of total delay is used in the synthesis of the speed controller. In the last stage of research time-frequency analysis was performed for: 1) reference current before and after filtration 2) load and motor velocity.

Keywords - digital filter; delay; multi-mass; spectral analysis; servomotors; chirp; frequency response; notch filter; motion control; mechanical vibrations; direct drive;

I. INTRODUCTION

Knowledge of the structure and parameters of the electric drive is important for tuning the speed controller. In many cases, the assumption of a rigid connection between the motor and the driven machine is insufficient. Most commonly used is the two-mass model representing the mechanical part of the drive [1]–[5]. In this article, structure of mechanical model was considered as four mass.

A common approach in industrial automation is implementation of additional filter of the reference current (torque), that ensures damping in these frequencies, where the torsional vibrations may occur [1], [6], [7]. To avoid stimulation of the resonance vibration, notch filters and biquad filters are applied [6], [8], [7].

The main goal of the research presented in this paper was to calculate delay of used filter. Simulation results are related to the exact parameters of the system, since these values are known. Information about the delay is necessary for the proper tune the controller. The control system characterized by long delays should have a slower controller otherwise system can be unstable. In high performance control system delay must be taken into account [1], [9].

II. MATHEMATICAL MODEL OF COMPLEX DRIVE SYSTEM

A. Torque control

Modern control methods of the electric drives allow to simplify the power electronic subsystem to the combination of a first-order inertial element and a transport delay, as it is shown in equation (1):

\[ H_{cur}(s) = \frac{k_T}{1 + s \cdot \tau_{cur}} e^{s \cdot \tau_{sim}} \]  

(1)

Taking into consideration the mentioned simplification, only the mechanical part of the drive system is worth to be analyzed more carefully. This part may be depicted, in a general case, as a multi-mass system [10]. In this paper four-mass system was considered. Transition delay of the torque control loop was assume \( \tau_{sim} = 0.3 \) ms and inertia of the torque control loop \( \tau_{cur} = 0.2 \) ms. Torque constant is equal to \( k_T = 17.5 \) Nm/A. PMSM (permanent-magnet synchronous motor) was considered.

B. Four-mass mechanical system

In this article has been considered a four-mass system with one motor generating torque \( T_M \) and one load torque \( T_L \) on opposite side. Schematic diagram of the four mass mechanical system was shown in Fig. 1. Equations of motions (2) were rewritten to state-space model (3). In the simulation tests transfer functions models were used, which were connected as shown Fig. 2. The derivation of multi-mass mechanical model was described in paper [10].
\[
\begin{align*}
J_y \ddot{\theta}_y + b_1(\dot{\theta}_y - \dot{\theta}_i) + k_1(\theta_y - \theta_i) &= T_u \\
J_y \ddot{\theta}_y + b_2(\dot{\theta}_y - \dot{\theta}_i) + k_1(\theta_y - \theta_i) - b_2(\dot{\theta}_y - \dot{\theta}_i) - k_2(\theta_y - \theta_i) &= 0 \\
J_y \ddot{\theta}_y + b_3(\dot{\theta}_y - \dot{\theta}_i) + k_1(\theta_y - \theta_i) - b_3(\dot{\theta}_y - \dot{\theta}_i) - k_2(\theta_y - \theta_i) &= 0 \\
J_y \ddot{\theta}_y - b_2(\dot{\theta}_y - \dot{\theta}_i) - k_1(\theta_y - \theta_i) &= -T_u
\end{align*}
\]

(2)

Mainly the delay was caused by the filters and \( f \) determines \( f_{\text{max}} \). The sampling time is equal to \( \mu_0s10 \) and don't excite system to vibrate. Vibrations in the system are attenuated due to self-damping. The structure of speed controller and method of selection of the control parameters (8)-(9) are based on work \[11\]. The controller is robust to changes of object parameters. Other type of robust or adaptive controller can also be applied \[12\], \[13\]. Moment of inertia can change in range < \( J_{\text{min}} \); \( J_{\text{max}} \) > and estimate torque constant can change in range < \( k_{T_{\text{min}}} \); \( k_{T_{\text{max}}} \) >. The article focuses on the calculation of the value of summary delay \( \tau_{d0} \). Mainly the delay was caused by the filters and the torque control loop. Limit level should be set according to the location of resonance and anti-resonance. In this perspective, it is clearly visible, that the occurrence of resonance is related with location of poles of \( H_{1,1}(s) \), while the location of zeros determines the anti-resonance. Simulations were performed with parameters presented in Table 1.

**TABLE 1. PARAMETERS OF SIMULATION MODEL**

<table>
<thead>
<tr>
<th>( \phi (\text{deg}) )</th>
<th>( f_{\text{ar}} )</th>
<th>( \xi_{\text{ar}} )</th>
<th>( f_{\text{r}} )</th>
<th>( \xi_{\text{r}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.753</td>
<td>1</td>
<td>10</td>
<td>105</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>153</td>
<td>5</td>
<td>251</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>10</td>
<td>350</td>
<td>90</td>
</tr>
</tbody>
</table>

C. Structure of Control System

The structure of the control system consists of a cascade connection of the current controller and speed controller (Fig. 3). Motor shaft position was measured using an incremental sensor. The sensor allows the measurement of 512000 pulses/rev. This is very important because the direct motor is modeled without gear. The maximum motor speed is equal to 2.5 rev/s. Obtained measurement velocity is equal to \( \tau_s = 100 \mu s \). The output of the speed controller is the current command in q-axis \( i_{\text{ref}}^{q} \). The high mechanical resonance frequency components are transmitted by the control system. Use of the anti-resonance filter allows to attenuate the frequency components. As a result the current command is filtered \( i_{\text{ref}}^{q, f} \) and don’t excite system to vibrate. Vibrations in the system are attenuated due to self-damping. The structure of speed controller and method of selection of the control parameters (8)-(9) are based on work \[11\]. The controller is robust to changes of object parameters. Other type of robust or adaptive controller can also be applied \[12\], \[13\]. Moment of inertia can change in range < \( J_{\text{min}} \); \( J_{\text{max}} \) > and estimate torque constant can change in range < \( k_{T_{\text{min}}} \); \( k_{T_{\text{max}}} \) >. The article focuses on the calculation of the value of summary delay \( \tau_{d0} \). Mainly the delay was caused by the filters and the torque control loop. Limit level should be set according to the location of resonance and anti-resonance. In this perspective, it is clearly visible, that the occurrence of resonance is related with location of poles of \( H_{1,1}(s) \), while the location of zeros determines the anti-resonance. Simulations were performed with parameters presented in Table 1.
to the parameters of the motor and the power electronic inverter. Used parameters were as follows:

\[ k_{\text{fmin}} = k_{\text{fmax}} = k_f, \quad \text{current limitation } i_q = \pm 6 \, A, \]

\[ J_{\text{min}} = 0.9 \cdot J_{\Sigma} \text{ and } J_{\text{max}} = 1.1 \cdot J_{\Sigma}. \]

\[ K \approx 0.5 \cdot \frac{\pi}{2\tau_{\text{dly}}} \cdot \frac{J_{\text{min}}}{k_{\text{fmax}}} \quad (8) \]

\[ T \approx 2 \cdot \frac{J_{\text{max}}}{K \cdot k_{\text{fmin}}} \quad (9) \]

D. Excitation signals

Identification can be performed in several approaches (Fig. 4). Generally all method are divided into two main groups of identification algorithms: parametric and non-parametric. Use of parametric approach requires selection of model structure and fitting algorithm. It is used for auto tuning of controller. Non-parametric method allows researcher for further signal analysis in domain of: time, frequency or time and frequency. Nonparametric identification in frequency domain was used in this paper.

Two different identification excitation signals are utilized: chirp signal and pseudo-random binary sequence signal.

Chirp signal is a cosine wave with linear frequency modulation. Frequency is linearly changed from \( f_{\text{min}} \) to \( f_{\text{max}} \) in time equal to \( \tau_{\text{inc}} \). Chirp signal is described by equation:

\[ x(k) = \cos \left( 2\pi \left( f_{\text{min}} + \frac{f_{\text{max}} - f_{\text{min}}}{2\tau_{\text{inc}}} k \cdot \tau_{\text{inc}} \right) k \cdot \tau_{\text{inc}} \right) \quad (10) \]

The waveform and spectrum of this signal are shown in Fig. 5. As it is visible, all the frequencies in the range \( <f_{\text{min}}, f_{\text{max}}> \) are present with equal magnitude in the spectrum.

Second commonly used signal is pseudo random binary sequence (PRBS). The algorithm is widely used in cryptography because allows repeatable generation of random binary signal. Feedback used in shift register of 11 length occurs on the 9 and 11 position [14]. This signal contains all frequency and is simple in implementation because it require only two signal level. PRBS signal in time and frequency domain are shown in Fig. 6.

In first step of identification both excitation signals \( i_q^{\text{ref}} \) were given to model and velocity \( \omega \) were measured. Sampled data were transformed from time domain to frequency domain using algorithm radix-2 of Fourier transform. Obtained frequency response data (Fig. 7 and Fig. 8) easily allow to calculate mechanical resonance frequency. Exact calculation of the values of the parameters can be obtained using one of the following methods: read values from spectrum, use of additional algorithm to read automatically the parameter from spectrum [15]–[17] or use one of parameters identification methods.

III. DELAY OF ANTI RESONANCE FILTER

A. Delay of filter

Determination of the filter delay requires knowledge of the phase characteristics. Calculation of the amplitude and phase characteristics for each type of filter can be done using (11) and (13). Spectral transmittance can be obtained by applying \( s = j\omega \) in transfer function. Then the obtained expression is separated into real and imaginary part according to the formula (11).

\[ H(j\omega) = P(\omega) + jQ(\omega) \quad (11) \]

where \( P(\omega) = \text{Re}\{H(j\omega)\} \) and \( Q(\omega) = \text{Im}\{H(j\omega)\} \).
The amplitude characteristics in a logarithmic scale was obtained using (12). However phase characteristic was obtained using (13).

\[ G_{\log}(\omega) = 20 \log(|H(j\omega)|) = 20 \log \sqrt{P^2(\omega) + Q^2(\omega)} \]  
\[ \phi(\omega) = \arg(H(j\omega)) = \arctg \frac{Q(\omega)}{P(\omega)} \]  

Another issue is the delay. Knowing phase characteristic delays can be easily determined. Delay of filter can be described using phase delay (14) and group delay (15).

\[ \tau_p(\omega) = -\frac{\phi(\omega)}{\omega} \]  
\[ \tau_{gr}(\omega) = -\frac{d\phi(\omega)}{d\omega} \]  

Group delay is a derivative of the phase of the pulse. If phase characteristic is linear then delay will be constant. However phase is nonlinear for considered filters. Therefore delays are different for each frequency.

Group delay was calculated for analog filter. However in the control system digital filters were used. The use of the prewarping method and high sampling rate according to attenuation frequency allows to have the same amplitude and phase characteristics of the digital filter [6].

**B. Notch filter**

The most commonly used band-stop filter which cuts off resonance frequency from signal [8], [16], [18]. It requires accurate identification of mechanical resonance frequencies [3], [8]. In the case of multiple resonances, use the appropriate number of notch filter. Filters are connected in series. Each filter cause additional delay, which should be added to \(\tau_{d0}\) and taken into account in the synthesis of the speed controller.

The bandstop filter (notch filter) may be described with the following transfer function [6]:

\[ T_F(s) = \frac{s^2 + \omega_2^2}{s^2 + s \frac{\omega_2}{Q} + \omega_2^2} \]  

Such a filter is characterized by two parameters: \(\omega_2\) – frequency of attenuation and \(Q\) – inverse of width of the attenuation. The discrete form of this filter is presented in [6].

To determine the phase characteristics of the filter \(T_F(j\omega)\) was separated in real and imaginary part (17). \n
\[ T_F(j\omega) = \frac{(j\omega)^2 + \omega_2^2}{(j\omega)^2 + j\omega \frac{\omega_2}{Q} + \omega_2^2} = \frac{\omega_2^2 - \omega^2}{\omega_2^2 - \omega^2 + \frac{\omega_2}{Q} - j} \]  
\[ = \frac{\left(\omega_2^2 - \omega^2\right)^2 - \left(\omega_2^2 - \omega^2\right) \left(\frac{\omega_2}{Q}\right) + \left(\frac{\omega_2}{Q}\right)^2}{\left(\omega_2^2 - \omega^2\right)^2 + \left(\frac{\omega_2}{Q}\right)^2} j \]  

Substitution of the real and imaginary part of the notch filter to the expression (13) allowed to obtain filter phase characteristics (18).

\[ \phi(\omega) = \arctg \left( \frac{\omega_2}{Q(\omega^2 - \omega_2^2)} \right) \]  

Usage of equation (15) and knowledge of (18) allowed to determine group delay (19).

\[ \tau_{gr}(\omega) = \frac{\left(\omega_2\right)^2 + \omega_2^2}{Q(\omega^2 - \omega_2^2) + \left(\omega_2\right)^2} \]  
\[ = \frac{Q_0\omega_2}{Q\left(\omega^2 - \omega_2^2\right) + \omega_2^2} \]  

Was proposed simplified formula (20) for future applications. This delay was used to set speed controller parameters.

\[ \tau_{gr}(\omega = 0) = \frac{1}{Q_0\omega_2} \]  

First notch filter attenuating mechanical resonance frequency was analyzed. Two cases of filter parameters were considered: a) \(f_r = 105\) Hz and \(Q = 0.6\), b) \(f_r = 105\) Hz and \(Q = 3.33\). For both cases were shown phase and group delay in Fig. 9 and Fig. 10 respectively. The filters have a different width notch. According to (20) increasing the notch width will cause more delay by the bandstop filter for low frequencies from \(f = 0\) Hz to the frequency of first edge of barrier. Reducing the width of the notch allows to reduce the delay (measured as response time) but the filter has more oscillatory step response Fig. 10. Additionally excessive narrowing of the notch may lead to insufficient attenuation of the mechanical resonance frequency.
C. Bi-quad filter

Another filter which is used to suppress the mechanical resonance frequencies is the inverted resonant characteristics filter. The filter is described by equation (21) which cancel one resonance block (7). Discrete form is presented in [6].

$$T_p(s) = \prod_{i=1}^{L} \frac{s^2 + 2 \zeta_{i} \omega_{i} + \omega_{i}^2}{s^2 + 2 \zeta_{i} \omega_{i} s + \omega_{i}^2}$$

(21)

Fig. 12 shows the delay for the filter of the parameters allowing to damp mechanical resonance frequency of the first component described by equation (7). The disadvantage of this solution is to strengthen the antiresonant component in the load speed $\omega_L$ and the negative group delay.

D. Low pass filter - Inverse Chebyshev

The classic approach to suppress mechanical resonance frequencies component is the use of a low pass filter. Filter cut-off frequency is chosen below the first mechanical resonance frequency [8]. Best filtration properties can be obtained when the first zero of filter is matched to the first resonant frequency. This ensures maximum attenuation of the first resonance. This paper considers the Inverse Chebyshev approximation as low pass filter. The selection of filter parameters is presented in [6]. The general form of the filter is expressed by (22) and (23) [6].

$$T_p(s') = G \cdot T_0 \prod_{i=0}^{L} \frac{s'^2 + |z_i|^2}{s'^2 - 2 \text{Re}(p_i) s' + |p_i|^2}$$

(22)

$$T_0(s') = \begin{cases} 1, \text{ even } N \\ - \frac{p_i}{s' - p_i}, \text{ odd } N \end{cases}$$

(23)

The filter has been chosen to suppress the first mechanical resonance. The delay of the filter (Fig. 13) is greater than in the previously described solutions.

IV. RESULTS

Taking into account the delay contributed by common filters used to suppress mechanical resonance frequency in the current command only notch filter was considered in further results. Two sets of parameters were selected: $Q_1 = 0.6$ and $Q_2 = 3.33$. Notch frequencies for the three notch filters selected on the basis of identification are given: 105 Hz, 251 Hz and 350 Hz. Notch filter cause a longer delay characterized by a greater attenuation than the filters having smaller delay (Fig. 15). In the results Fig. 16 - Fig. 18 and Fig. 19 - Fig. 21 shows respectively: the use of the low pass filter effect on the measured speed $\omega_f$, the effect of resonance suppression filter on the current command $i_{ref,f}$, the speed of the motor $\omega_M$ and load speed $\omega_L$. Important is the lack of vibration on the load speed $\omega_L$ side and fast control time. Speed set point is equal to 0.5 rev/s (Fig. 14). At time equal to 0.25 s was applied torque impact on load side equal $T_L = 15$ Nm (Fig. 14). At time equal to 0.6 s was added constant load torque equal $T_L = 15$ Nm. In both cases $Q_1$ and $Q_2$ the control system operates properly. However increasing the delay results in change of controller parameters which lead to worse dynamics. After adding constant load torque dynamic of system is poorer for the case $Q_1$ (Fig. 18) than $Q_2$ (Fig. 21).

Fig. 22 - Fig. 25 shows time-frequency analysis of reference current before and after filtration for both conditions. Also velocity of motor and load were analyzed (Fig. 26 - Fig. 29). Comparing current spectrograms can be notice that higher frequency components (251 Hz and 530 Hz) were attenuated, however some artefact left for lower frequency equal to 105 Hz. Especially for $Q_2$, which gives better dynamic than $Q_1$, poorer attenuation of lower frequency leads to oscillation of load velocity Fig. 29.
V. CONCLUSIONS

The paper presents effect of use filter that suppress mechanical resonance frequency components in current command $q$-axis in direct drive. Direct drive and executive part of the system was modeled as four mass which characterized by three resonant frequencies.

Paper shows how to identify the mechanical resonance frequencies using nonparametric approach. Two excitation signals were used to gather the data used for spectrum calculation. Exact calculation of parameters value can be obtain using one of follow method: read values from spectrum, use of additional algorithm to read automatically the parameter from spectrum [15], [16] or use one of parameters identification methods [4], [19], [20]. Proper identification allows the proper synthesis of filter which attenuate mechanical resonance components carried by the speed control. Both exciting signals chirp and PRBS give very good identification results. Considered filters are as follow: notch, reversed resonance characteristics filter and inverse Chebyshev low pass filter.
Presented equations allows for calculation of filter group delay caused by notch filter. Paper presents delay introduced by filter: reversed resonance characteristics filter and inverse Chebyshev low pass filter. Booth filter are tuned for first resonance block. Determine delay is taken into account during the synthesis of the robust speed controller.

The tests show that increasing the filter delay negatively affects the dynamics of the control system (Fig. 28 and Fig. 29). The delay time should be short. However reducing the time delay causes less attenuation of mechanical resonance band (Fig. 29). As a result filter with a very low delay is narrow. Narrow band attenuation may be inadequate to the task of suppressing mechanical resonance. Using this method, user must decide whether he want to good dynamics or good attenuation for wide bandwidth. Use of this method for low-frequency resonance is troublesome because of the increasing delay time (20). Obtained results confirm that using more narrow notch filter allows to increase the dynamics of speed control loop.

Performed time-frequency analysis of simulation result (Fig. 22 - Fig. 29) confirms that applying notch filter for higher mechanical resonance frequencies (251 Hz and 350 Hz) method give good attenuation and small delay (high dynamics). However for lower mechanical resonance frequency (105 Hz) designer of control system has to choose between dynamics and attenuation of oscillations. Control system with Q2 is more likely to oscillate after load change than for case Q1 (Fig. 29). Comparing filtered currents (Fig. 24 and Fig. 25) both have smooth shape in time domain, however level of artefact for mechanical resonance frequency is higher in case Q2.

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REFERENCES