A 6-port Network Technique for Extraction of 2-Port DUT Noise Correlation Matrix: A Theoretical Verification through Modeling and Simulation

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***Abstract* — In this paper, we demonstrate the 6-port network technique for the extraction of noise correlation matrix of a 2-port DUT and thus its noise parameters. The passive 6-port network which is extended to an 8-port is designed using five Wilkinson power dividers, and a 90° hybrid. Three Low Noise Amplifiers (LNA) as well as SP4T switch are incorporated into the now 8-port network structure, components which are all modeled in Advanced Design System (ADS). The equations of the proposed technique are applied to the noise spectral density from the output of the 8-port network, which enable the extraction of the noise correlation matrix as well as the noise parameters of a 2-port DUT. Comparison of the noise correlation matrix and noise parameters of AFM02N5 transistor, in ADS library, obtained from the simulation of the 6-port network technique, with that from direct device simulation shows perfect agreement; thereby validating the proposed technique.**

Keywords* — *modeling, simulation, 6-port network, 8-port network, calibration, noise figure, noise parameters, noise waves correlation matrix.

I. Introduction

Noise parameters refer to a set of four parameters that completely characterize the noise performance of a given 2-port device under test (DUT). The four noise parameters consist of two real parameters, the noise resistance R_n and minimum noise figure NF_{min}; and the real and imaginary parts of a complex parameter, known as the optimum source admittance Y_{opt} [1].

The noise parameters of a linear two-port (DUT) have for a long time been determined using the impedance tuner technique [2]. Escotte. *et al.* [3] has provided quite a review and comparison of various impedance tuner-based techniques for the extraction of the noise parameters of a linear 2-port DUT. In this technique, essentially, the impedance tuner is used to vary the source impedance seen from the DUT, and various noise figures corresponding to the various source impedances can be recorded, from which the minimum noise figure, $NF_{\min}$ is found and thus the corresponding admittance at which this minimum occurs, known as the optimum admittance, Y_{opt}. The remaining noise resistance R_n can be obtained by substitution when the noise figure at 50 ohms impedance is known; leading to the complete determination of the noise parameters. The drawbacks of this method which have been cited in the literature include accuracy problems, high cost, long measurement time, and especially the inconvenience of employing bulky and heavy tuners [3]-[5].

Several alternative methods, not requiring the use of tuners, have emerged which are usually accompanied by very complex calibration procedures. Works such as [6] - [7] employ frequency and time-domain analysis to extract the noise parameters. Others such as [8], to cite but a few, apply some numerical approximations to a variation of the multi-impedance method to extract the two port noise parameters of a given DUT. There are still other works such as [9], [10] in which the relationship between the noise wave correlation matrix and noise parameters forms the basis of the measurement system. The Six-port network technique [11] was suggested in [12]-[13] as a means of extracting the noise wave correlation matrix of a DUT. The authors have experimentally demonstrated the use of the 8-port extension of the 6-port network for the extraction of the noise correlation matrix as well as the noise parameters of a given 2-port DUT [14].

In this paper, we model the 6-port network technique for the extraction of the noise correlation matrix of a linear 2-port and thus its noise parameters in Advanced Design System (ADS) Computer Aided Design (CAD) software tool. The simulated output noise spectral density from the 8-port extension of the 6-port network which represent the combined effect of the noise waves due to the DUT and that due to the modeled 8-port network are subjected to the calibration equations leading to the determination of the noise waves exclusively due to the DUT, and thus its noise

parameters. The noise correlation matrix of a linear 2-port DUT, in ADS library, as well as its noise parameters can be obtained by direct device simulation, which can then be compared with that obtained from the simulated 8-port technique for the same device. Perfect agreement was achieved between the two results, thus validating the 8-port technique for noise parameters extraction.

II. THEORY OF THE 8-PORT TECHNIQUE

Referring to Fig. 1 (a) and (b), the relationship between the output noise wave spectral density, **b** at the output ports of an 8-port network and the associated noise waves sources, **a** from terminations and noise waves sources internal to the 8-port network, **c** are given by:

$$\begin{pmatrix} \mathbf{b}_e \\ \mathbf{b}_s \\ \mathbf{b}_i \end{pmatrix} = \begin{pmatrix} \mathbf{S}_{ee} & \mathbf{S}_{es} & \mathbf{S}_{ei} \\ \mathbf{S}_{se} & \mathbf{S}_{ss} & \mathbf{S}_{si} \\ \mathbf{S}_{ie} & \mathbf{S}_{is} & \mathbf{S}_{ii} \end{pmatrix} \begin{pmatrix} \mathbf{a}_e \\ \mathbf{a}_s \\ \mathbf{a}_i \end{pmatrix} + \begin{pmatrix} \mathbf{c}_e \\ \mathbf{c}_s \\ \mathbf{c}_i \end{pmatrix} \tag{1}$$

Here, subscripts e is used for output ports (ports 1~ 4), i for input ports (ports 7 and 8), and s for externally applied noise source ports (ports 5 and 6), respectively. Thus the S-matrix represents the partitioned S-parameters of the 8-port network. It has been shown [14] that, when a 2-port DUT is connected to the 8-port network, the output noise spectral density vector taking account of the noise waves from a DUT, can be derived from (1) and expressed as

$$\mathbf{b}_e = \mathbf{M}\mathbf{a}_e + \mathbf{P}\mathbf{a}_s + \mathbf{Q}\mathbf{c}_i + \mathbf{\Lambda}\mathbf{c}_D + \mathbf{c}_e \tag{2}$$

$\mathbf{b}_e$ then represent the noise wave vector delivered to the output ports (ports 1 to 4) and $\mathbf{c}_D$ represents the noise wave due solely to the DUT. **M**, **P**, **Q** and **Λ** are embedded matrices derived from the partition matrices in (1). $\mathbf{a}_e$ and $\mathbf{a}_s$ are noise waves coming from terminations and externally applied noise source respectively and their contributions can be known since they are thermal noise sources. $\mathbf{c}_i$ and $\mathbf{c}_e$ which are also correlated need to be determined from calibration, leaving $\mathbf{c}_D$ then as the only unknown to be determined.

From (2), the output noise wave spectral density from the 8-port network which is due solely to the DUT is given by:

$$\begin{aligned} \boldsymbol{\beta}_D &= d(\mathbf{\Lambda}\mathbf{C}_{DD}\mathbf{\Lambda}^\dagger) \\ &= \mathbf{n}_D - d\begin{Bmatrix} \mathbf{M}\mathbf{M}^\dagger + \mathbf{P}\mathbf{P}^\dagger + \\ \left(\mathbf{C}_{ee} + \mathbf{Q}\mathbf{C}_{ie} + \mathbf{C}_{ei}\mathbf{Q}^\dagger + \mathbf{Q}\mathbf{C}_{ii}\mathbf{Q}^\dagger\right) \end{Bmatrix} \end{aligned} \tag{3}$$

$\mathbf{n}_D$ is the output noise wave spectral density of the entire 8-port network, when the DUT and external noise sources are connected as shown in Fig. 1 (b), normalized by kT_o. $\boldsymbol{C}_{DD}$ is then the desired normalized noise wave correlation matrix of the DUT. The noise wave vector of the eight-port network incident directly on the output ports, normalized by kT_o is given by

$$\mathbf{C}_{ee} = \mathbf{n}_{e,av} - \begin{bmatrix} d\left\{\mathbf{S}_{ee}\mathbf{S}_{ee}^\dagger + \mathbf{S}_{es}\mathbf{S}_{es}^\dagger + \mathbf{S}_{ei}\mathbf{S}_{ei}^\dagger\right\} - \\ \left(|S_{11}|^2 \quad |S_{22}|^2 \quad |S_{33}|^2 \quad |S_{44}|^2\right)^t \end{bmatrix} \tag{4}$$

$\mathbf{n}_{e,av}$ is the average output noise wave spectral density when the input ports are terminated by 50 ohm impedances. S_{ij} are S-parameters of the 8-port network. The remaining correlations are determined as

$$\begin{aligned} & d\left(\mathbf{Q}\mathbf{C}_{ie} + \mathbf{C}_{ei}\mathbf{Q}^\dagger + \mathbf{Q}\mathbf{C}_{ii}\mathbf{Q}^\dagger\right) = \\ & \mathbf{n}_T + \begin{pmatrix} |M_{11}|^2 \\ |M_{22}|^2 \\ |M_{33}|^2 \\ |M_{44}|^2 \end{pmatrix} - d\left\{\mathbf{C}_{ee} + \mathbf{M}\mathbf{M}^\dagger + \mathbf{P}\mathbf{P}^\dagger\right\} - d(\mathbf{\Lambda}\mathbf{C}_T\mathbf{\Lambda}^\dagger) \end{aligned} \tag{5}$$

where $\mathbf{n}_T$ is the average output noise wave spectral density when the input ports are alternately terminated by an *open*, *short* and *offset open* impedances. These three scenarios give the simultaneous equations required to determine the various components of the correlation. For every such cases, only one port (port 7 or 8) is such terminated, while the other (port 7 or 8) is terminated in 50 ohm impedance. The normalized noise correlation matrix for such 50 ohm termination is given by

$$\mathbf{C}_T = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

Thus, the normalized noise correlation matrix of the DUT, $\boldsymbol{C}_{DD}$ can be determined from

$$\boldsymbol{\beta}_D = d(\mathbf{\Lambda}\mathbf{C}_{DD}\mathbf{\Lambda}^\dagger) \tag{7}$$

Since the DUT S-parameters are known, the noise wave correlation matrix in (7) can be converted into the noise current correlation matrix $\mathbf{C}_Y$ by applying the transformations in [15] as

$$\mathbf{C}_Y = Y_o(\mathbf{I} + \mathbf{Y})\mathbf{C}_{DD}(\mathbf{I} + \mathbf{Y})^\dagger \tag{8}$$

where **Y** is the normalized Y-parameters of the DUT and Y_o represents the reference admittance of 1/50 Simens. The

ABCD noise correlation matrix $\mathbf{C}_A$ are then obtained using the conversion formula

$$\mathbf{C}_A = \mathbf{T}_{AY}\mathbf{C}_Y\mathbf{T}_{AY}^{\dagger} \tag{9}$$

The transformation matrix $\mathbf{T}_{AY}$ is given by

$$\mathbf{T}_{AY} = \begin{pmatrix} -y_{11} & 1 \\ -y_{21} & 0 \end{pmatrix}^{-1} \tag{10}$$

Here y_{ij} are elements of $\mathbf{Y}$.

The noise parameters are then computed from the formulae in equations (11)-(13), where a_{ij} are the elements of the ABCD noise correlation matrix obtained from the transformations in [15].

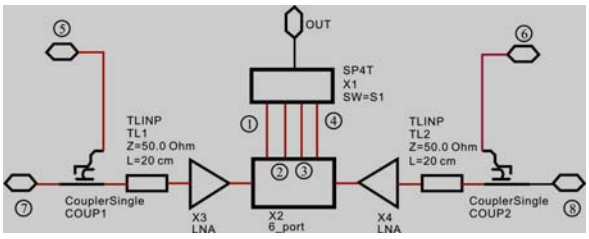
$$R_n = \frac{a_{11}}{4} \tag{11}$$

$$Y_{opt} = \sqrt{\frac{a_{22}}{a_{11}} - \left\{\mathrm{Im}\left(\frac{a_{12}}{a_{11}}\right)\right\}^2} + j\,\mathrm{Im}\left(\frac{a_{12}}{a_{11}}\right) \tag{12}$$

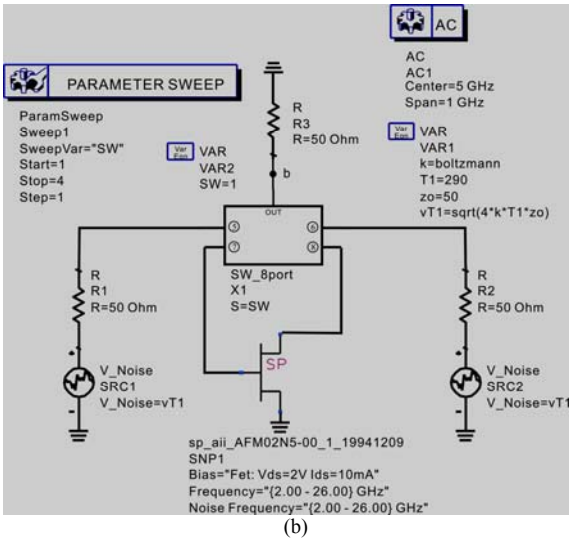
$$NF_{\min} = 1 + \frac{a_{12} + a_{11}Y_{opt}^*}{2} \tag{13}$$

III. Verification of the Theory

To ascertain the validity of the formulation presented in section II, all the components in the 8-port network were modeled as schematics in ADS, a CAD tool by Agilent Technologies® (now Keysight Technologies®). Since the purpose of the circuit simulation is to verify the formulation presented in section II, we modeled the components using circuit elements in ADS rather than the measured S-parameters and noise parameters of the 8-port network components and of the DUT.



(a).



(b)

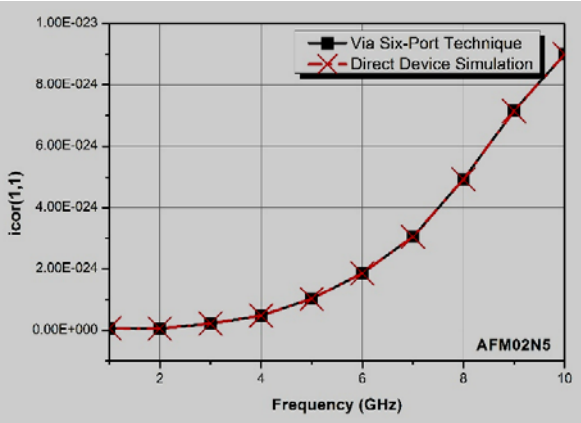
Fig. 1 ADS Schematics (a) Schematic for the 8-port network and (b) schematic for DUT noise parameter measurement.

The conventional Wilkinson power dividers and 90°-hybrid employed in the six-port network structure were designed using transmission line **TLINP** in ADS. The transmission lines were set to generate noise at room temperature, T_o. The LNAs were modeled using the component **AMP** in ADS. The typical values of the gains in the datasheet were used for the parameters of **AMP**. The two coaxial connection lines used to connect the LNA and 13 dB directional coupler were also modeled using **TLINP** in ADS. The SP4T switch was modeled using the ADS component **SPDT_dynamic**. The sub-circuit **SP4T** in Fig. 1(a) was formed using the modeled SP4T switch and the modeled LNA. Thus, the sub-circuit **SP4T** includes the SP4T and LNA. A pair of 13 dB directional couplers was modeled using the component **CouplerSingle** in ADS, shown in Fig. 1 (a). The circuit of Fig. 1 (a) was then converted into the sub-circuit **SW_8port** shown in Fig. 1 (b).

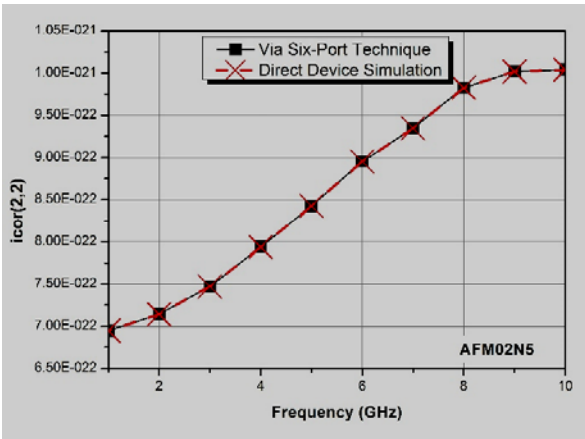
Firstly, two 50 ohm resistors which generate thermal noise were connected in place of the DUT shown in Fig. 1 (b). The noise powers at the output port for a parameter sweep of **SW** were computed using voltage **b** across noiseless resistor **R3** in Fig. 1 (b). The variable **SW** represents the SP4T states. Using a set of computed voltages **b**, the noise wave vector $\mathbf{c}_e$ can be obtained in accordance with (4).

The noise powers can be computed directly using the simulated voltages in the circuit simulation. The external noise source with a given ENR was implemented by varying the temperature of the noise source **V_noise** in Fig. 1 (b).

Similarly, alternately connecting 3 terminations such as *short*, *open*, *offset open* to ports 7 and 8 with the other port terminated by 50 ohm termination,



(a)



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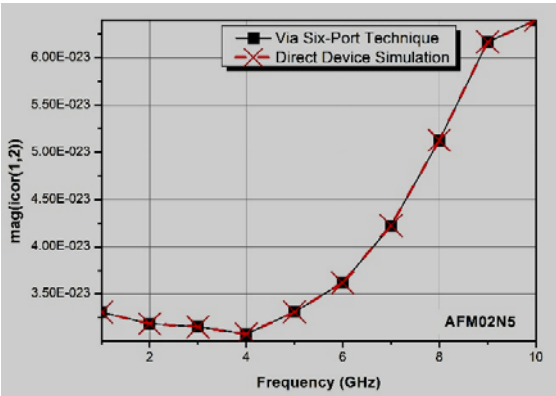

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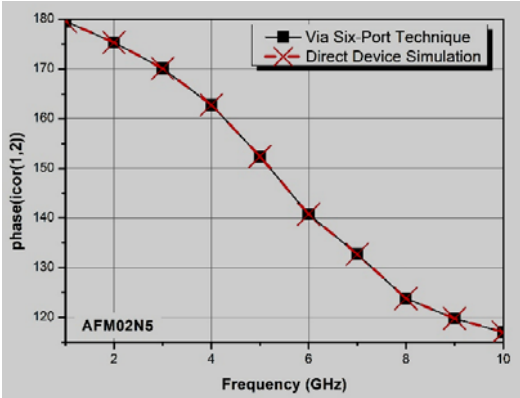


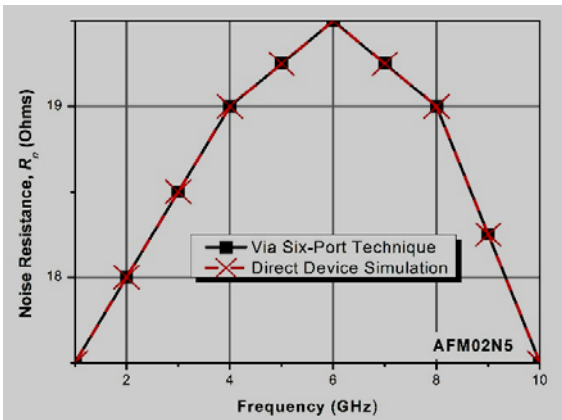
(d)

Fig.2 Comparison of the extracted current noise correlation matrix of AFM02N5 transistor in ADS library based on the proposed method with that obtained from direct device simulation: (a) self-correlated noise power spectrum at port 1 of DUT, **icor (1,1)** (b) self-correlated noise power spectrum at port 2 of DUT, **icor (2,2)** (c) magnitude of cross-correlated noise power spectrum of DUT, mag(**icor (1,2)**) and (d) phase of cross-correlated noise power spectrum of DUT, phase (**icor (1,2)**).

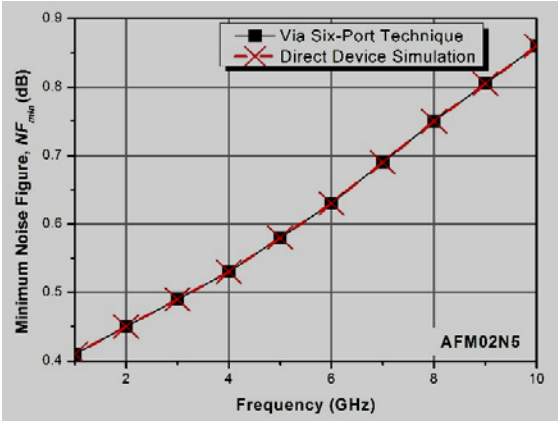
the noise vector $\mathbf{c}_i$ and its correlation with $\mathbf{c}_e$ could be computed using the simulated noise powers and previously determined $\mathbf{c}_e$.

Equation (5) in section II was used in the display window of ADS to compute the noise vector $\mathbf{c}_i$ and its correlation with $\mathbf{c}_e$.

The DUT is then, connected as shown in Fig. 1 (b). The selected DUT is the active device, **AFM02N5** in the ADS transistor library and is employed as shown in Fig. 1 (b). The output noise voltages **b** for a sweep of the parameter **SW** can then be obtained. Using the previously determined calibration data, the excess noise powers for the selected DUT alone were computed. Equations (7) to (9) are then used to compute the current noise correlation matrix (**icor** (x, y)) shown in Fig. 2 and equations (11) to (13) are used to compute



(a)



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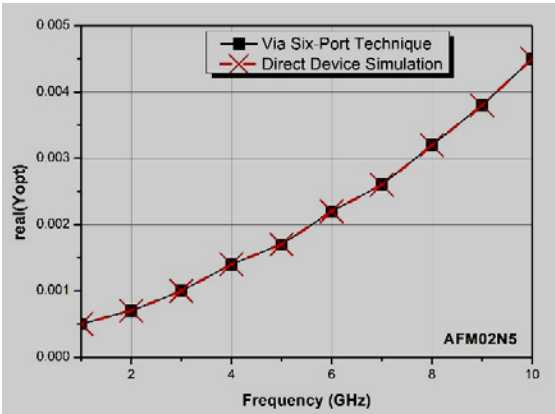

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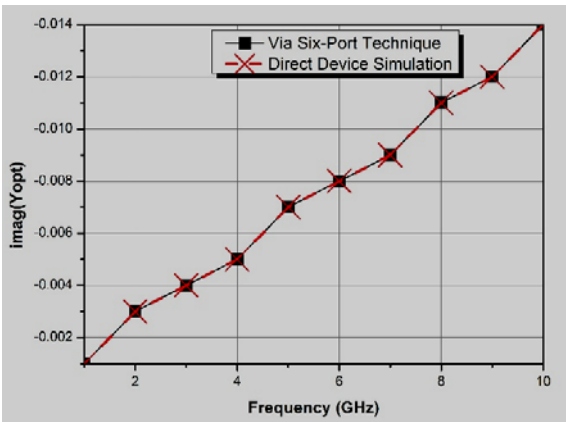

(d)

Fig. 3 Comparison of the extracted noise parameters based on the proposed method with thenoise parameters for AFM02N5transistor in ADS library: (a) Minimum noise figure, NF_{min} (b) Noise resistance, R_n (c) Real part of Optimum Admittance, real(Y_{opt}) and (d) Imaginary part of Optimum Admittance, imag(Y_{opt})

the noise parameters in the ADS display; shown in Fig. 3. Note that the noise correlation matrix as well as the noise parameters for the selected DUT can be obtained directly from **S-parameter simulation** in ADS. This allows for comparison with the 6-port network technique.

Figure 2 shows the comparison of the extracted noise current correlation matrix of **AFM02N5** transistor found in ADS library, and the perfect agreement between the two results requires no commentary. Figure 3 show the extracted noise parameters of **AFM02N5** in ADS library based on the proposed technique which is compared with that from direct device simulation. As is obvious from the perfect agreement between the two simulation results, the proposed technique for the extraction of noise parameter is conclusively verified, both in terms of calibration and DUT measurement.

IV. Conclusion

A formulation for the extraction of the noise wave correlation matrix using an 8-port extension of the 6-port network was presented. The formulation as well as the calibration procedures was verified through simulation. Theoretically, the 8-port extension of the 6-port network operates for a frequency band of 2~8 GHz. The 6-port network technique produced results of noise correlation matrix as well those of noise parameters for sample DUT with remarkable accuracy; thus conclusively validating the proposed technique.

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