A Nonlinear Adaptive State-Observer for Pressurized Water Reactors

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Abstract - Nuclear plant safety as well as availability can be quite well improved if such functions as early fault diagnoses and on-condition maintenance are automated. To realize these functions, on-line monitoring of nuclear reactors is required, and state observation is one of the key and basic techniques of reactor-state monitoring. Moreover, state-observation is also the basis of applying advanced reactor control strategies. State observation deals with recovering desired states from available measurements. Pressurized water reactor (PWRs) are complex dynamic systems with high nonlinearity and strong uncertainty, which leads to the necessity of developing nonlinear adaptive reactor observation technique. Thus in this paper we propose for PWRs a nonlinear adaptive observer with only the measurements of both the nuclear power and reactor inlet coolant temperature. It is proved theoretically that this new observer is globally convergent and robust to the modeling and parameter uncertainties and exterior disturbances by a well-designed adaptation law. Numerical simulation results in both the cases of large-range load increase and decrease show the feasibility and high performance of this new nonlinear adaptive observer.

Keywords - Nonlinear adaptive observer, nuclear power, PWR

I. INTRODUCTION

State observation method deals with recovering desired state variables of a dynamic system from available measurements. State-observation has already been widely applied in the fields of fault detection [1-4], operational monitoring [5, 6, 11] and constructing dynamic output feedback control laws [7-14] for nuclear reactors. The most widely used state observers in practical engineering are Kalman filter (KF) and extended Kalman filter (EKF). Kalman filter has been widely used in the areas of monitoring and fault detection and isolation [1-4] and dynamic output-feedback stabilization [8, 9] for nuclear reactors. Particle filter (PF) is also a promising state-observation strategy which has been applied to nuclear reactors successfully [5, 6]. However, KF is only applicable to linear systems, and EKF is based on the linearized model. Since nuclear reactors are highly complex and nonlinear, the dynamical features of the original and the linearized model is quite different, it is necessary to design nonlinear observers for nuclear reactors. Shtessel designed a nonlinear reactor state observer by sliding mode technique, which was then applied to form a dynamic output-feedback power-level controller for space reactor TOPAZ II [10]. Based upon the idea of feedback dissipation, Dong et al. gave a dissipation-based high gain filter (DHGF) for the state-observation of pressurized water reactors (PWRs) [11, 12], and applied the DHGF to construct the dynamic output-feedback control [12, 13]. Since there must be uncertainties induced by the modeling error, parameter perturbation and exterior disturbances, an adaptive nonlinear observer was proposed for PWRs [14].

The above nonlinear observers need the measurements of the nuclear power as well as both the outlet and inlet coolant temperatures of a given reactor. However, in the practical engineering, there may be fault in outlet coolant temperature sensor. Therefore, it is very necessary to give a nonlinear observer without the measurement of the outlet coolant temperature. In this paper, a novel nonlinear adaptive state-observer with only the measurements of both the nuclear power and reactor inlet coolant temperature is proposed for PWRs. It is proved that this newly-built observer is not only globally convergent but also robust to dynamic uncertainties. Numerical simulation results verify the correctness of the theoretic results and also show the high performance of this newly-built observer.

II. PROBLEM FORMULATION

Based on the point kinetics with one equivalent delayed neutron group and the reactivity feedback given by the variations of average fuel and coolant temperatures, and based on the energy balance of the reactor thermal-hydraulic loop, the PWR-like reactor dynamics for observer design can be written as

\[
\begin{align*}
A\dot{\eta}_i &= [\rho_i - \beta + \alpha_i (T_i - T_{in}) + \alpha_t (T_{out} - T_{in})]n_i + \beta c_i, \\
\dot{c}_i &= \lambda(n_i - c_i), \\
\mu_i \dot{T}_i &= -\Omega(T_i - T_{in}) + P_n n_i, \\
\mu_i \dot{T}_{out} &= \Omega(T_i - T_{in}) - 2M (T_{out} - T_{in}), \\
\dot{\rho}_i &= G_i n_i, 
\end{align*}
\]

where \(n_i\) is the relative nuclear power, \(c_i\) is the relative concentration of delayed neutron precursor, \(\rho_i\) is the reactivity due to the control rods, \(\beta\) is the fraction of delayed neutrons, \(A\) is the effective prompt neutron lifetime, \(\lambda\) is the effective radioactive decay constant of delayed neutron precursor, \(\alpha_t\) and \(\alpha_c\) are the reactivity coefficients of the fuel and coolant.

DOI 10.5013/IJSSST.a.17.35.5

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ISSN: 1473-804x online, 1473-8031 print
temperatures respectively, $T_i$ is the average temperature level, $T_{av,m}$ and $T_{av,in}$ are respectively the average temperature of the coolant inside the core and the temperature of the coolant entering the core, $P_0$ is the rated power level, $\mu_f$ is the total heat capacity of the fuel, $\mu_r$ is the total heat capacity of the reactor coolant, $\rho_i$ is the reactivity given by the control rods, $G_i$ is the reactivity worth of control rods, and $v_i$ is designed speed signal of control rods, i.e. the control input. It is worthy to be noted that $a_t$ is guaranteed to be negative by reactor design.

Define the deviations of $n_t$, $c_t$, $T_o$, $T_{av,m}$, $T_{av,in}$ and $\rho_t$ from their steady values, i.e. $n_{oc}$, $c_{oc}$, $T_{oc}$, $T_{av,oc}$, $T_{av,in}$ and $\rho_{oc}$ as

$$\delta n_t = n_t - n_{oc},
\delta c_t = c_t - c_{oc},
\delta T_o = T_o - T_{oc},
\delta T_{av,m} = T_{av,m} - T_{av,oc},
\delta T_{av,in} = T_{av,in} - T_{av,in},
\delta \rho_t = \rho_t - \rho_{oc}.\quad (2)$$

Moreover, for the convenience of expressing the complete nonlinear state-space model, define

$$x = \left[ \delta n_t, \delta c_t, \delta T_o, \delta T_{av,m}, \delta T_{av,in} \right]^\top,
\xi = \delta z_r = G_i^\top \delta \rho_t,
\nu = v_t,$$  
where $\delta z_r$ is the total displacement of the control rods.

In the practical engineering, $x_t$ i.e. $\delta T_{av,m}$ can also be obtained from measurement directly. However, there usually exists large delay effect in obtaining $\delta T_{av,in}$. Moreover, there may exist the case that the temperature sensors is in error, and cannot be utilized for measurement any more. Therefore, here, we only adopt $x_t$ i.e. $\delta n_t$ as the system output for observer design.

From the above definition and discussion, the nonlinear state-space model for observer design can be written as

$$\dot{x} = f(x, \delta T_{av}) + g(x)(\xi + \theta),
\dot{\xi} = u,
y = h(x),$$  
where

$$f(x, \delta T_{av}) = \begin{bmatrix}
-A^{-1}(\lambda(x_t - x_i) + A^{-1}\left(\mu_f(n_{oc} + x_i) - \mu_r\right)
-\mu_f\Omega(x_t - x_i) + \mu_r\Omega
-\mu_f\Omega(x_t - x_i) - 2\mu_r M(x_t - \delta T_{av})
\end{bmatrix},$$
$$g(x) = \begin{bmatrix}
A^\top G_i(n_{oc} + x_i) & 0 & 0
\end{bmatrix},
h(x) = x_i,$$
and $\theta$ is the exterior reactivity disturbance or the parameter uncertainties corresponding to $a_t$, $\alpha$ and $G_i$. Here, it is assumed that $\theta$ is a constant.

From the above modeling and analysis, it is practically necessary and academically meaningful to give an adaptive observer approach with satisfactory performance.

Suppose that the observer takes the form as

$$\begin{cases}
\dot{x} = f(\hat{x}, \delta T_{av}) + g(\hat{x})(\hat{\xi} + \hat{\theta}),
\dot{\hat{\xi}} = u - k_{oc}(n_{oc} + x_i)e_t,
\end{cases}$$  
where

$$e_t = [e_1, e_2, e_3, e_4]^\top = [\hat{x}_1 - x_1, \hat{x}_2 - x_2, \hat{x}_3 - x_3, \hat{x}_4 - x_4]^\top.\quad (11)$$

The design result about the gains and the adaptation law of $\hat{\theta}$ so that $\dot{\hat{x}}$ can converge to reactor state $x$ globally.

\section{III. ADAPTIVE STATE-OBSERVER DESIGN}

The design result about the gains and the adaptive law of observer (10) is summarized as following Theorem 1 which is the main result of this paper.

\textbf{Theorem 1.} Consider nonlinear system (6) with observer taking the form as (10). Design observer gain matrices $K_O$ and the adaptation law of $\hat{\theta}$ as

$$K_O = \begin{bmatrix}
k_{oc} + A^{-1}(\alpha_i x_i + \alpha_f \hat{x} + G_i \hat{\theta})
2\lambda
-\mu_f P_i (k_{oc} (n_{oc} + x_i) - 1)\end{bmatrix},$$
$$\hat{\theta} = -k_{oc}(n_{oc} + x_i)e_t.\quad (13)$$
where $k_{oc}$, $k_{oc}$, and $k_{oc}$ are given positive constants. Then, observer (10) with the gain matrix satisfying (12) and adaptive law satisfying (13) provides globally asymptotic reactor state observation for system (6).

\textbf{Proof:} Based upon nonlinear state-space model (6) and observer dynamics (10), the observation error dynamics is governed by differential equations

$$\dot{e}_t = \lambda (e_t - e_{t-1}) - 2\lambda e_t,$$
$$\mu e_{t-1} = -\Omega (e_{t-1} - e_t) + P k_{oc} (n_{oc} + x_i)e_t,$$
$$\mu e_\xi = -\Omega (e_\xi - e_t) - 2\mu e_t + \alpha_i \hat{x} + \alpha_f \hat{x} - k_{oc} (n_{oc} + x_i)e_t,$$
$$\hat{e}_t = -k_{oc}(n_{oc} + x_i)e_t,$$
where

$$e_\xi = \hat{\xi} - \xi,$$  
$$\hat{\theta} = \hat{\theta} - \theta.$$
(19)

Choose the Lyapunov function candidate for the above observation-error dynamics as
\[ V_c(e, e_t, \dot{\theta}) = \frac{A}{2} e_i^2 + \frac{\beta}{2\lambda} e_i^2 + \frac{\mu}{2\mu_{CR}} (\mu e_i e_t + \mu_{CR} e_t) + \frac{G}{k_{CR}} e_i^2 + \frac{1}{2k_{CR}} \dot{\theta}^2, \]

from which it is clear that \( V_c \) is not only positive-definite but also radially unbounded, which means proper definition. Then, by differentiating \( V_c \) along the trajectory given by observation error dynamics (14) – (18), we have
\[ \dot{V}_c = A e_i e_{ti} + \frac{\beta}{2\lambda} e_i e_{ti} + \frac{\mu}{2\mu_{CR}} (\mu e_i e_{ti} + \mu_{CR} e_{ti}) + \frac{G}{k_{CR}} e_i e_{ti} + \frac{1}{k_{CR}} \dot{\theta}. \]

From equation (22), it is clear that observation error \( e \rightarrow 0 \) as \( t \rightarrow \infty \), which means globally convergent observation of reactor state \( x \) can be obtained. This completes the proof of this Theorem.

Remark 1. From equations (22) and (18), it is clear that observation errors \( e \) and \( e_t \) enter the set given by
\[ x = [e, e_t, e_i, e_{ti}, \dot{\theta}]^T, \]

Moreover, from equation (14) that \( e_i=0 \) (i=1, 4) then
\[ \left( G_{i,k_{i}C} + k_{i}C_{o} \right) \int_{0}^{t} (s_{in} + x_{in}) e_{i} d\tau + \dot{\theta} = 0. \]

For a given uncertainty \( \theta \), we can see that from (24) that if gains \( k_{i}C_{o} \) or \( k_{o}C_{o} \) is larger, then term \( \int_{0}^{t} (s_{in} + x_{in}) e_{i} d\tau \) is smaller, which induces a smaller observation error \( e_i \). Thus, larger \( k_{i}C_{o} \) and \( k_{o}C_{o} \) gives a stronger observer robustness.

IV. SIMULATION RESULTS WITH DISCUSSIONS

In this section, observer (10) will be applied to solve the power-level problem with the only measurements of both the nuclear power and inlet coolant temperature for a nuclear heating reactor (NHR). The NHR is a new type of nuclear power reactor developed by the Institute of Nuclear and New Energy Technology (INET) of Tsinghua University. The NHR has inherent safety performance determined by its advanced design features such as the integral arrangement, self-pressurizing, entire-range natural circulation, hydraulic driving control rods and passive residual heat removing [15, 16]. The NHR can be applied to the areas such as electricity production, district heating and seawater desalination. Since the NHR is a typical light water small modular reactor (SMR) which can be used to constitute micro-grids coupled with those renewable energy resources such as the wind and photovoltaic. The NHR power-level should tightly follow the load given by environment temperature in the case of district heating and by fresh-water demand in the case of sea-water desalination.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Symbol</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.0069</td>
<td>( \alpha )</td>
<td>-3.85e-4 (1/C)</td>
</tr>
<tr>
<td>( \dot{\lambda} )</td>
<td>0.08 (1/s)</td>
<td>( G_{r} )</td>
<td>0.0048</td>
</tr>
<tr>
<td>( \mu )</td>
<td>588.544 (kW/(C))</td>
<td>( M )</td>
<td>394.89 (kW/(C))</td>
</tr>
<tr>
<td>( \mu_{c} )</td>
<td>25151 (kW/(s/C))</td>
<td>( \Omega )</td>
<td>125.68 (kW/(C))</td>
</tr>
</tbody>
</table>

The simulation model of the NHR is composed of the point neutron kinetics with six delayed neutron groups and lumped parameter models of reactor thermal-hydraulics, primary heat exchanger, U-tube steam generator (UTSG), feed water pump, pipes and volume cells [17]. Here, the UTSG water-level controller adopts that one presented in [18]. The parameters of the NHR at the middle of the fuel cycle in 100% full power-level (FP) are given in Table 1. In this simulation the power-level control adopts the PD control given in [19], and the feedback loop is shown by Figure 1. From Figure 1, the observer is used to reconstruct the average coolant temperature information based on the measurements of both the nuclear power and inlet coolant temperature, the observed average coolant temperature drives the power-level controller to generate proper control rod speed signal with the measured relative nuclear power together.

Moreover, the following two case studies are done in this simulation:

Case A (Large-Scale Load Lift): The load increases linearly from 20% to 100% FP in 60s linearly.

Case B (Large-Scale Load Reject): The load steps down from 100% to 20% FP.

Both cases A and B represent hard operations of the NHR. The responses of both the simulated and observed values of the relative nuclear power, average fuel temperature \( T_f \) and outlet coolant temperature of the reactor core \( T_{out} \) as well as the designed value of control rod speed signal \( v_t \) in the cases of A and B are illustrated by Figs. 2 and 3 respectively. In this simulation, \( k_{ON}=10, k_{OS}^* = 0.01 \) and \( k_{OF}=0.01 \). The load signal is the demanded reactor power-level which gives the set points of the relative nuclear power and inlet coolant temperature. The variation of the load signal results in the setpoint variations of both the nuclear power and coolant temperature, which in turn enlarges the amplitude of error signals \( x_i \) defined by (3) and \( \delta T_{in} \) defined by (2). Furthermore, these two errors drive state-observer (10) to give the nearly asymptotic state-observation, which in turn drive the corresponding power-level control law to properly generate insertion or withdrawal control rod speed signal for eliminating the mismatch between the actual and demanded reactor power.

From both Figures 2 and 3, we can see that convergence of the state-observations corresponding to the key process variables, i.e. the relative nuclear power, average fuel temperature and outlet coolant temperature can be well
guaranteed, which is well in accordance with equation (22) that the observation error of reactor state $x$ is globally

Figure 1. Schematic view of the closed-loop system.

Figure 2. Simulation results in case A: (a) relative nuclear power, (b) average temperature of the fuel elements, (c) outlet coolant temperature of the reactor, and (d) control rod speed signal.
asymptotically convergent. Thanks to the satisfactory observation of the outlet coolant temperature, the power-level regulation performance is well provided. Moreover, from Figure 3, there exist sharp peaks in the dynamic responses of the nuclear power and average fuel temperature, which is caused by the sudden step of the load signal and large observation gains such as $k_{ON}$. Since observer (10) cannot response to the step of the load signal immediately, there must exist the sharp peaks. From the numerical simulation results, observer (10) can be utilized to construct the fault tolerant control law when the outlet coolant temperature sensor is in error.

Observer (10) can be applied to not only monitoring the operational state of a PWR but also constructing the fault-tolerant power-level control strategy. This observer can be easily implemented on those advanced digital control system platforms, and there are many experiments that should be done for verifying the state-observation performance in the practical engineering.

V. CONCLUSIONS

State-observation is the technique which reconstructs the internal or unmeasurable state-variables based on the system dynamic model the measurements, which is meaningful to monitoring the operational state and realizing the fault-tolerant control law for a nuclear reactor. Motivated by this, a nonlinear adaptive observer based on the measurements of both nuclear power and reactor inlet coolant temperature is proposed for PWRs, which provides globally convergent observation for the reactor state-variables, and whose robustness to the modeling error, the parameter uncertainties and the exterior disturbances is given by a well-designed adaptation law. Numerical simulation results not only verify the theoretic results but also show the feasibility and high performance of this newly-built nonlinear adaptive observer. The future works lies in verifying the performance of this observer by hardware-in-loop (HIL) simulation, and study the means of avoiding the peak phenomenon induced by high gain.

ACKNOWLEDGMENT

The work in this paper is jointly supported by Natural Science Foundation of China (NSFC) (Grant No. 61374045), Tsinghua University Initiative Scientific Research Program (Grant No.20121087992) and National S&T Major Project (Grant No.ZX06901).
The author would like to thank the anonymous reviewers for constructive comment. Moreover, the author would like to thank Prof. Huang Xiao-Jin deeply for consistent support, valuable discussions and constructive suggestions.

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