PID and Pole Placement Controller for Circular Flying Formation Satellite System

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Abstract — Currently the study of formation flying systems (FFS) and the control of satellite formation flying in circular orbit are becoming increasingly important. In this paper we analyse the circular orbit flying formation of a micro-satellite to assess its stability and control using pole placement, we then add a PID controller and a suitable actuator in order to decrease overshoot, improve steady state error response and settling time while optimizing transients to bring the satellite back into the ideal orbit. Simulation based design was carried out for a PID and pole placement controller for a satellite in circular earth orbit under inverse square gravitation using MATLAB software.

Keywords - PID, pole placement, controller, satellite, formation flying, control, simulation

I. INTRODUCTION

Nowadays control systems have widespread application in the leading and guidance, navigation, and control of rocket, missiles, satellites, spacecraft, airplanes as good as ships at sea and other numerous applications of these control systems exist all around us (Malik, Zaidi, Aziz, & Khushnood, 2001). Satellite dynamic and control are important subjects involving a various range of topics from mechanics and control theory. Also Control the altitude of a satellite is important especially the one used for worldwide communications in a geo-stationary orbit. The study of the motion of a satellite and stability on circular orbit is important in designing spacecraft and satellite control systems. A bunch of satellites requires operating automatically to accomplish the control requirements of a mission without human intervention.

Satellite formation flying has newly become an significant field of study and research in the space industry and there has been increasing attention in the research of formation flying system. It has rapidly developed into a most important area of activity and studies in space science, communication, defense and other related applications and due to the potential advantages over the conventional large size massive satellite, such as task flexibility, low cost, better observation efficiency, increased reliability and stability of orbit flying and improved survivability, many FFS are currently being proposed for two field of military and non-military (Thanapalan & Veres, 2005; Xincheng, Ying, Zhisheng, & Zhiyong, 2008) and control of satellites is the subject of much research efforts in the control systems community at huge using a diversity of configurations and control algorithms. (For example see these references, (De Queiroz, Kapila, & Yan, 2000; Mueller, 2001; Pongvithithum, Veres, Gabriel, & Rogers, 2005; Thanapalan & Veres, 2005; Yeh, Nelson, & Sparks, 2000) Problems such as steady motions and their stability have attracted the attention of many researchers, cf., (Wang, Lian, & Chen, 1995; Yan, Kapila, & Sparks, 2000).

The goal is to design a stable equilibrium orientation for the circular motion of a satellite.

In economic view, it’s not cheap if we place a big satellite with all its functions, into orbit path than some smaller ones of the same group weights orbit, and the capability of small satellites to fly in accurate formation has made a wide range of new applications possible. As a result as the number of missions and tasks that use spacecraft flying in formation, proposed and are under improvement, still increases, one can imagine assembly lines of standardized spacecraft and satellite, thus lowering the price of building them significantly (Topland, 2004).

Hughes and Wie are standard references on spacecraft and satellite dynamics, they suggested engineers a complete guide for analysis and function of modern spacecraft and satellite attitude dynamics. So from its roots in classical mechanics and confidence on stability theory, to the advancement of practical stabilization ideas during the past quarter of a century, Spacecraft Attitude Dynamics improve all aspects of the subject in a unified and coherent method (Hughes, 1986; Wie, 1998).

Concerning attitude control of spacecraft, Wie, Weiss and Arapostathis show that a PD controller stabilizes a spacecraft(Wie, Weiss, & Arapostathis, 1989). They use the classical spacecraft model with no moving parts. several works were established on the attitude control, where various approaches were used, and in particular PID regulators. Robert A. Pazm have proposed the PID controller is most commonly that used dynamic control technique (Robert, 2001). Then Several methods for tuning the controllers have been presented. Over 85% of all dynamic (low-level) controllers are of the PID variety and the aim of their work is to provide a brief overview of the PID controller. R. Nagarajan has submitted a simple adaptive predictive FL controller for attitude control of micro-satellite is developed and its performance compared with the conventional PID controller, after the analysis it is clear that the performance of the APFLC has better performance in terms of percentage overshoot and rise time (Ramachandran, Paulraj, Szazali, Zuriadah, & Hoh, 2008).
Mohammad Arfaa Malik used modeling orbit flying formation and the objective of this work was to evolve a design based on modeling and simulation of an orbit controller for a satellite orbiting into a circular orbit that was based on the dynamic model of flying satellite (Malik, et al., 2001). Yue Xincheng proposed, a parameter related Lyapunov function was used to design robust controller for a satellite formation flying system in the circular path around the Earth based on an uncertainty model derived from a nonlinear relative position equation (Xincheng, et al., 2008). Other references studied spacecraft attitude control using several reaction wheels as actuators (Hall, 2000; Hall, Tsiotras, & Shen, 1998; Linarelli & Wen, 1996; Tsiotras, Shen, & Hall, 2001). E. Elakkary, A. Echchaibi, submitted a control law by consecutive poles Placement, that they were interested in control of a satellite, that using wheels of reaction, through state feedback (Elakkary, Echchaibi, & Elalam, 2005).

Morton Pederson, Toplind developed a variety of nonlinear controllers to control attitude of a spacecraft using thrusters and reaction wheels as actuators. Linearization and Lyapunov theory is used to derive two linear and four nonlinear controllers. Three of the nonlinear controllers rely on cancellation of system nonlinearities, while the fourth is a sliding mode controller (Toplind, 2004). In some above references using reaction wheels as actuator and thruster are survived, and in other one using wheels of reaction, through state feedback and pole placement.

![Diagram of control system](image)

**Figure 1.** procedure of theory

This paper developed circular orbit Formation Flying of a micro-satellite state space of motion, and its stability schemes for controlling a satellite in the Inverse square Gravitation and preparing a design based on simulation of the control system and an circular orbit controller. Also Controllability and Stability of system are investigated, and PID is added to the previous work (Asadi, Gladamini, & Anbarani, 2011) in order to improve the system and combination of PID and pole placement controlling method for decreasing overshoot and settling time and improve its response and transient are submitted (Asadi, Mohammadi, & Gladamini, 2011; Asadi, Gladamini, et al., 2011, Kautsky, Nichols, & Van Dooren, 1985, Peng, Li, 2003, Ramamurthy, et al., 2008, Robert, 2001). The simulations generated with the help of "MATLAB" it may be very useful as an engineering tool in designing the on-board propulsion system and also for assessing the amount of fuel required for the specified life of the satellite Methodology and procedures of this work is shown in above figure 1.

II. SATELLITE RELATIVE POSITION DYNAMIC MODEL STATE SPACE EQUATION

In this part the Formation Flying of satellites theory system modeling is shown in below figure 2 and we have submitted mathematical modeling result and state space (Asadi, Gladamini, et al., 2011, Elakkary, et al., 2005, Hall, 2000, Hughes, 1986, Malik, et al., 2001, Wei, 1998).

An significant assumption of the development of the controllers is that the gravity of the Earth is the only external disturbance torque which affects the satellites.

While carrying out the mathematical modeling of the system, we consider the planar motion of an orbiting satellite in the inverse-square gravitational field of the earth. The gravitational force exerted on the satellite is \( F = -\frac{GMm}{r^2} \) and the satellite is approximated as a particle of mass \( M \). The satellite motion is more conveniently described with polar coordinates. In this situation, \( r \) is Earth of mass and \( r \) is the vector pointing from \( M \) to \( m \) and \( \theta \) is the distance from the center of the earth to the center of the satellite, and \( G \) is the universal gravitational constant. This system orbit in circular motion around a fixed center and thrusts are defined with respect to the tangential and radial directions according to the below figure 2. Equations (1, 2, 3) is result of modeling the system:

\[
\begin{bmatrix}
C_2 \\
C_3 \\
C_4
\end{bmatrix}
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 2\omega \cos \theta \\
0 & \frac{2\omega \sin \theta}{\lambda} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
\omega \\
n_1
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]  

(1)

\[\begin{bmatrix}
x \\
\omega \\
n_1
\end{bmatrix}
= \begin{bmatrix}
x = x_0 + C_2 \omega \cos \theta \\
\omega = \omega_0 + C_3 \omega \\
n_1 = n_1_0 + C_4 \omega \\
\end{bmatrix}
\]  

(2)

\[r(\theta) = \phi(\theta) - \phi = 0, \quad \phi = \lambda - \theta, \quad K = C^2 - \lambda^2 \]  

(3)

\[u_{10} = u_{20} = 0\]
It is noticeable to mention that this procedure not only show an absolute yes or no stability assessment, it can also illustrates information regarding the relative degree of stability of a stable system and help us to get final state space. For determination of linear system stability is to factor the characteristics equation and check the location of the roots.

\[
x(t) = A x(k) + B u(k), \quad k = 0, 1, 2, ...
\]

Now we assume that \( w = 2 \) and \( C = 2 \) in the state space. In other hand for locating satellite on the circular path \( r \) must be constant and \( \phi \) is variable. So for this reason, \( x_2 \) and \( x_4 \) must be: \( x_2 = r \cdot 0, x_4 = \phi \) in the output, thus we will have equation of matrix (6)a,b,c,d of main state space of orbit flying formation:

\[
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4
\end{bmatrix} =
\begin{bmatrix}
    0 & 1 & 0 & 0 \\
    12 & 0 & 0 & 8 \\
    0 & 0 & 0 & 1 \\
    0 & -2 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4
\end{bmatrix}
+ \begin{bmatrix}
    0 \\
    1 \end{bmatrix} u_1 + \begin{bmatrix}
    0 \end{bmatrix} u_2
\]

III. TRANSFER FUNCTION OF SYSTEM

A. Open Loop Transfer Function

In this part open loop system of circular orbit formation flying of a micro-satellite is proposed. Basic concept of open loop system is the output is simply commanded by input, there is no disturbances correction; also an open-loop control system uses an actuator as actuating tool to control the system process straight without using any feedback.

In this case the original transfer function is achieved in equation (7) and ideal Space transport by transfer function, transfer function of Sensor and actuator transfer function is shown in equations (8),(9),(10)

\[
G_f = \frac{4 s^5}{s^6 + 3 s^6 + 48 s^8 + 64 s^6}
\]

(7)

\[
G_i = \frac{8 s^3}{s + 3000}
\]

(8)

\[
G_s = \frac{8 s^5}{s^2 + 16 s + 64}
\]

(9)

\[
G_a = \frac{8 s^2 + 3 s + 1}{s^2 + s + 1}
\]

(10)

The total open loop transfer function of system is shown in below equation (11).

\[
G_{openloop} = \frac{96000 s^8 + 4032606 s^7 + 1196007 s^6 + 8448006 s^5}{s^{10} + 3020 s^9 + 60144 s^8 + 432560 s^7 + 1682600 s^6 + 6569000 s^5 + 1.441e607 s^4 + 2.843e607 s^3 + 4.994e607 s^2 + 6.223e607 s^1 + 6.146e607 s^0 + 4.915e607 s^9}
\]

(11)

B. Total Transfer Function of Close Loop

In this part Closed Loop Systems for this satellite is proposed, that has better accuracy and better control of transient and steady-state response, also they are Less sensitive to noise and disturbances, plant Variations, and Complex & expensive. Equation (12) shows close loop transfer function and it's clear that system has 4 poles in right area.

\[
G_{closedloop} = \frac{96000 s^8 + 4032606 s^7 + 1196007 s^6 + 8448006 s^5}{s^{10} + 3020 s^9 + 60144 s^8 + 432560 s^7 + 1682600 s^6 + 6569000 s^5 + 1.441e607 s^4 + 2.843e607 s^3 + 4.994e607 s^2 + 6.223e607 s^1 + 6.146e607 s^0 + 4.915e607 s^9}
\]

(12)

Step response has been investigated, in order to observing the stability. Step response is the time behavior of the outputs of a general system when its inputs increase from zero to one in a very short time and it is a standard for understanding that the system is stable or not, in other hand all of systems step response help us to monitor the
output of system cause knowing how the system responds to a sudden input is important.

According to Figure 3 it's clear that the system is unstable cause when its input change from zero to one in a very short time the amplitude of output tend to infinity.

IV. CONTROLLABILITY OF SYSTEM

In this part the controllability of system is surveyed because the system must be controllable in order to design controller. Controllability is so important for system and it is a significant feature of a control system, and the controllability feature has a serious role in many control problems and systems, such as stabilizing unstable systems by feedback which help to improve it, or optimal control. This process not only produces an complete stability analysis, it just show us that we can hope to control it and it also gives us information regarding the relative degree of stability of a stable system.

Here the controllability matrix is, $MC = [BB AB A2B A3B]$, Since rank $Mc = 4$, the system is controllable.

A second test for controllability that can be utilized in finding out controllability of the system with divide inputs is as follow. Equation (13) shows that a system is controllable if and only if

$$\text{Rank} \begin{bmatrix} I & A & A^2 & A^3 \end{bmatrix} = 4 \quad i = 1,2,3,4$$  \hspace{1cm} (13)

This satisfies the controllability criterion. So the system is controllable with tangential thrust. Thus it is possible to stabilize the system with only thrust in the tangential direction. So the matrix is full rank and the system is controllable.

V. POLE PLACEMENT ORBIT CONTROLLER DESIGN

In this part we have designed a pole placement in order to stabilize the system. Suitable actuator for this system which is shown in part III has been used that can be thruster. Then pole placement and experiment and error for finding best place have been used for decreasing the overshoot and improve the response and transient stability of system and we found out that the band width oscillation decreases by moving the poles to right side location, so the result is improved with the less oscillation. Suitable state feedback that is the most significant feature of modern control system and its various gain can stabilize Unstable systems and improve damping of oscillatory systems.

Pole placement design lead to placing the poles in this system at specific locations, it prepare the system to be controllable and stabilize them. It's important that the closed-loop transfer function poles are the values of $A-BK$. Again matrix $K$ that placement these poles to the desired locations in the plane can be computed by the place function.

This need high gain, that in turn makes the whole closed-loop structure so sensitive to disorder and distribution. The aim of design is to find the gain matrix $K$ such that the equation for the controlled system is the same to the ideal equation.

In this method, it is assumed that all state variables are accessible for feedback. Complex poles will result in a sinusoidal oscillation, the terms will tend to zero and will be stable if the real roots are negative, in other hand terms that increase dramatically and will become unstable if the system has positive roots. The rule is that if any roots are located on the right hand side of the plane makes a system unstable. In this case we would like to transfer the system poles to the left hand side of the plane in order to stabilizing system and keep the system stable.

The desired poles are $[-34,-37,-40,-45,-50+5i,-50-5i,-55,-60+1i,-60-1i,-65,-70,-72,-74,-76,-78,-80]$, so we used acker to transfer the poles to desire location in left side of the plane.

Figure 4: simulation result of first series of desire poles location

The new gains of feedback are

\[
\begin{align*}
    k &= 1.0e+029 \\
    &-0.0000 0.0000 0.0000 0.0000 0.0000 \\
    &0.0000 0.0000 0.0000 0.0000 0.0000 \\
    &0.0001 0.0022 0.0361 0.4075 2.861 9.4077
\end{align*}
\]

Figure 4 shows simulation result of first series of desire poles location. It's clear that the system has a quite big over shoot because of some zero which are in transfer function, thus we should decrease the over shoot of simulation result in order to getting more ideal design.

The band width oscillation decreases by moving the poles to new right side location, so in order to design we can follow above procedure.

As it is clear the system has overshoot and the settling time is long and it has a small steady state error in order to improve the system and remove above problem we can add PID controller to pole placement controller.

VI. PID CONTROLLER

The PID controller is the earlier controller consists of three basic modes, which are the Proportional mode, the Integral and the Derivative modes.
The PID controller can minimize the measured procedure value with a reference set point value. The difference that is error, is used to calculate a new procedure input. This input will try to modify the measured process value back to the desired set point. The structure of a PID control system is shown in figure 5.

As it is clear, although the system has a very small steady state error but the settling time is quite long. In addition the system has a big overshoot which is not suitable for our system. Integral mode can be used for making zero steady state error. Derivative modes can be used as the mode which can eliminate or decrease the overshoot for having feasible orbit flying system.

In order to improve the system and remove above problem we can add PID controller to pole placement controller. the PID equation can be shown in equation (14)

\[
PID: \frac{K_p + K_i s + K_d s^2}{s} = \frac{K_p s^2 + K_i s + K_d}{s}
\]

With using Ki=80, Kp=50 and Kd=2 and multiplying PID equation to previous pole placement transfer function equation we can able to improve the system completely.

![Figure 5: The structure of a PID control system](image)

![Figure 6: Step response of adding PID controller to Pole placement controller](image)

and decrease overshoot, steady state error response and make short settling time to 0.6 second and adjust transient to bring back the satellite into the ideal orbit. the figure 6 shows the result of adding PID controller to Pole placement controller and as it shown the system has become ideal.

**VII. DISCUSSION AND RESULT**

After checking the controllability of the system it has been ascertained that there is a way to stabilize the satellite, it would be dealt by use of the state-feed back control and chose suitable Actuator for maintaining the satellite into its desired or assigned orbit. Pole placement method has been successfully used in the design of output feedback flying formation satellite, it has moved the poles to desired location in order to stabilizing the system for better response and keep the system on the circular path. Simple design principle which is by assigning closed loop poles on desired location, making the closed loop system (under control) faster and stable. It wants to guarantee system with enough damping effects on oscillations over wide range of loading conditions.

We have assigned poles to desire location in Matlab software and it results the stable response. According to figure 4, the simulation result of desire poles location system had a quite big over shoot because of some zero which are in transfer function, thus we should decrease the over shoot of simulation result in order to getting more ideal design. In order to decrease over shoot and oscillation we could transfer the location of the poles to right side more than, thus that the bandwidth oscillation decrease dramatically by moving the poles to right side but in the negative area and closer to axis on the right side of previous poles. Simple design principle which is by assigning closed loop poles on desired location, making the closed loop system faster and stable. It will guarantee a system with enough damping effects on oscillations over wide range of loading conditions. According to the simulation results, it improves system dynamic stability. A particular advantage of this method is that no freedom in the design parameters is lost. Only problem with this technique is that more states are required to be known. These states can be either measured or estimated. Other reason for using pole placement is that some controller cannot control the spacecraft’s attitude without the thrusters like byapunov controller.

The system has a very small steady state error but the settling time is quite long. In addition the system has a big overshoot which is not suitable for our system. In this part we have added the PID controller to the system in order to improve the system and remove above problem.

By multiplying PID equation to previous pole placement transfer function equation we could able to improve the system completely and decrease overshoot, steady state error response to zero and make short settling time to 0.6 second and adjust transient to bring back the satellite into the ideal orbit. According to figure 6 by applying PID method the system is improved and optimized.

One main advantage stands out above the rest when using a PID controller. And it can control various systems or devices with little human interaction and it also allows many processes to run at once. Also two PID controllers could be used together to make better dynamic performance.
VIII. CONCLUSION

The current work has tried to design and simulate the orbit controller for the satellite orbiting into a circular orbit in the Inverse square Gravitation by adding PID orbit controller to pole placement method for a circular satellite orbiting around the earth in the Inverse square Gravitation. The benefits of combination of these two method are decrease overshoot, steady state error response and making short settling time and optimizing transient response to bring back the satellite into the ideal orbit and guarantee the orbiting. On the other hand pole placement method can move the poles to desired location in order to stabilizing the system for better response and keep the system on the circular path and they are easy to implement and it make the closed loop system faster and stable.

REFERENCES